# Econ 210B
# Discussion Session N°6

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# Plan for Today

Midterm Review

# Problem A: Environmental Quality and Flow Policies

**Context:** The problem considers a social planner maximizing an objective function that depends on consumption and environmental quality. The evolution of environmental quality follows a dynamic equation, and the planner chooses consumption over time.

**Objective Function:**

$$\max_{c(t)} \int_0^\infty e^{-\rho t} \left[\sigma \ln c(t) + (1-\sigma) \ln S(t)\right] dt$$

where $\sigma \in (0,1)$, $\rho > 0$, and

$$\dot{S}(t) = \left(1 - \frac{S(t)}{2}\right) S(t) - c(t), \quad c(t) \geq 0, \quad S(t) \geq 0.$$

# Problem A: (a) Hamiltonian Formulation

**Question:** Write the *current value* Hamiltonian associated with the optimization problem for the planner, and indicate the state variable and the control variable(s). Define the co-state variable $\mu(t)$.

# Problem A: (b) Optimality Conditions

**Question:** Obtain the optimality conditions for the control, the co-state, and the state variables. Explain how you can determine the transversality condition. Provide an economic interpretation of the co-state variable.

$$\max_{c(t)} \int_0^{\infty} e^{-\rho t} \left[\sigma \ln c(t) + (1-\sigma) \ln S(t)\right] dt$$

$$\dot{S}(t) = \left(1 - \frac{S(t)}{2}\right) S(t) - c(t), \quad c(t) \geq 0, \quad S(t) \geq 0.$$

# Problem A: (c) Steady State Analysis

**Question:** Define the steady state $S^* > 0$ such that $\dot{S} = 0$. Characterize the steady state. Does it always exist? Is it unique? Can the steady state be $S^* > 2$? Explain and show your work.

# Problem A: (d)-(e) Decentralized Version

**Setup:** The decentralized version corresponds to a representative household maximizing the same objective function but with $Z(t)$ instead of $S(t)$.

**Objective Function:**

$$\max_{c(t),x(t)} \int_0^\infty e^{-\rho t} \left[\sigma \ln c(t) + (1-\sigma) \ln Z(t)\right] dt$$

The system evolves as:

$$\dot{S}(t) = \left(1 - \frac{S(t)}{2}\right) S(t) - x(t), \quad c(t) = x(t)(1-\tau(t)) + T(t).$$

**Question:** Write the *present value* Hamiltonian and derive the optimality conditions. Define the co-state variable $\theta(t)$.

# Problem A: (f) Planner vs Decentralized Outcomes

**Question:** Compare the optimality conditions of the decentralized version and the social planner. How do they differ? In which case is $S$ in steady state higher or lower? Explain.

# Problem A: (g) Policy Intervention in Decentralization

**Question:** Can a tax (or subsidy) $\tau(t)$ align the decentralized solution with the social planner's allocation? Why or why not?
Use the condition $T(t) = \tau(t)x(t)$ to ensure budget balance.

# Problem B: Lucas Trees in Finite Time

**Context:** The problem considers an economy with two agents, $i = \{A, B\}$, who live for two periods ($t = 1, 2$).
The economy has two trees that produce stochastic output, and agents trade assets competitively in markets.
Both agents maximize expected present value of lifetime utility with logarithmic preferences and discount factor $\beta \in (0, 1)$.

**Stochastic Output:**

$$\text{Tree 1: } y^1(s) = 1 + s, \quad \text{Tree 2: } y^2(s) = 2(1 - s),$$

where $s$ takes values 0 or 1 with equal probability.

**Notation:**

- Agents trade assets at market prices $P_t^1$ and $P_t^2$.
- Denote the holdings of Tree 1 by agent $i$ at the beginning of time $t$ as $z_{1,t}^i$, and for Tree 2 by $z_{2,t}^i$.
- The total existing stocks for each tree are normalized to 1.

# Problem B: (a) Budget Constraint

**Question:** Write the $t = 1$ and $t = 2$ budget constraints for agent $i$.

# Problem B: (b) Lagrangian and Optimality Conditions

**Question:** Write the Lagrangian for agent $i$ and obtain the corresponding optimality conditions.

# Problem B: (c) Competitive Equilibrium Conditions

**Question:** State the conditions for a competitive equilibrium in this economy, ensuring all market clearing conditions are included.

# Problem B: (d) Initial Stock Allocation and Market Prices

**Setup:** Suppose initial stock holdings are:

$$z^A_{1,1} = 1, \quad z^A_{2,1} = 0, \quad z^B_{1,1} = 0, \quad z^B_{2,1} = 1.$$

**Question:** Characterize the stock prices and consumption allocations. Are agents able to fully insure? Explain.

# Problem B: (e) Alternative Initial Stock Allocation

**Setup:** Suppose initial stock holdings are:

$$z_{1,1}^{A} = z_{2,1}^{A} = z_{1,1}^{B} = z_{2,1}^{B} = 0.5.$$

**Question:** Will stock prices differ compared to part (d)? Why or why not? Explain and show your work.