Econ 210B
Discussion Session N°11: Exam Review

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Plan for Today

Final 2024 Problem B: OLG Production Economy

Review on Search

PS4 Problem B: On The Job Search

Final Tips on Aiyagari

Final 2024 Problem B: OLG Production Economy

$$s_t^t \geq -\underbrace{\phi \frac{1}{R_t}}_{b} w_{t+1}^t \qquad s_t^t \geq 0$$

- OLG economy where individuals live for three periods: young, adult, and old.
- When young and adult individuals are endowed with one unit of time they supply (labor) inelastically.
- The size of each generation is normalized to 1 and constant, so the total population is 3.
- Individuals can save by holding capital or lend/borrow via a competitive credit market.
  - Young individuals can borrow up to a fraction $\phi \in [0, 1]$ of the present value of their labor income as adults.
  - Adults and old individuals cannot borrow.
- $u(c) = \ln c$, $\beta = 1$

$c_t^t \qquad c_{t+1}^t \qquad c_{t+2}^t$

$$F(K,N) = K^{\alpha} N^{1-\alpha}$$

$$F(K,1) = K^{\alpha} \qquad / : N$$

$$f(k) = k^{\alpha}$$

$$w = F_N = f(k) - r f'(k)$$

$i \setminus t$	1	2	3	4		
-1	c_1^{-1}					
0	c_1^{0}	c_2^{0}				
1	c_1^{1}	c_2^{1}				
2		c_2^{2}				

Final 2024 Problem B: OLG Production Economy

- Output in the economy is produced by a representative firm with a constant returns to scale technology:

$$Y = K^{\alpha} N^{1-\alpha},$$

  where K is aggregate capital, and N is total labor supplied.
- Markets for factors are perfectly competitive. $\longrightarrow$ $w = F_N$ $\quad R = F_K$
- The ownership of initial capital K_0 is equally split between initial adults and initial old.

(a) Write the maximization problem for generation t in this economy. Derive all the optimality conditions. Define a competitive equilibrium for the economy, making sure to include all the relevant market clearing conditions.

(b) Without solving fully for the equilibrium, can you tell from the optimality conditions and the market clearing conditions whether the young individuals will ever borrow in equilibrium? If your answer depends on parameters, what are the key parameters that determine whether borrowing will take place? Please discuss.

(a) Maximization Problem

Define the maximization problem for generation t, build the Lagrangian.

$$\max_{\{c_t^t,\, c_{t+1}^t,\, c_{t+2}^t,\, s_t^t,\, s_{t+1}^t\}} \quad \ln c_t^t + \beta \ln c_{t+1}^t + \beta^2 \ln c_{t+2}^t \qquad \beta = 1$$

s.t.

$$(1) \quad c_t^t + s_t^t \leq w_t$$

$$(1) \quad s_t^t \geq -\phi \frac{1}{R_t} w_{t+1}$$

$$>(2) \quad c_{t+1}^t + s_{t+1}^t \leq w_{t+1} + R_{t+1} s_t^t$$

$$(2) \quad s_{t+1}^t \geq 0$$

$$(3) \quad c_{t+2}^t \leq R_{t+2} s_{t+1}^t$$

$$\mathcal{L}: \ln c_t^t + \ln c_{t+1}^t + \ln c_{t+2}^t + \lambda_t \left[w_t - c_t^t - s_t^t \right] + \mu_t \left[\phi \frac{1}{R_t} w_{t+1} + s_t^t \right]$$
$$+ \lambda_{t+1} \left[w_{t+1} + R_{t+1} s_t^t - c_{t+1}^t - s_{t+1}^t \right] + \mu_{t+1} \left[s_{t+1}^t \right] + \lambda_{t+2} \left[R_{t+2} s_{t+1}^t - c_{t+2}^t \right]$$

(a) Maximization Problem

$\{c_t^t, c_{t+1}^t, c_{t+2}^t, s_t^t, s_{t+1}^t\}$

CS $\lambda_t, \lambda_{t+1}, \lambda_{t+2}, \mu_t, \mu_{t+1}$

Derive FOC.

$$\mathcal{L}: \ln c_t^t + \ln c_{t+1}^t + \ln c_{t+2}^t + \lambda_t\left[w_t - c_t^t - s_t^t\right] + \mu_t\left[\phi \frac{1}{R_t} w_{t+1} + s_t^t\right]$$
$$+ \lambda_{t+1}\left[w_{t+1} + R_{t+1} s_t^t - c_{t+1}^t - s_{t+1}^t\right] + \mu_{t+1}\left[s_{t+1}^t\right] + \lambda_{t+2}\left[R_{t+2} s_{t+1}^t - c_{t+2}^t\right]$$

$\{c_t^t\}$ $\quad \dfrac{1}{c_t^t} = \lambda_t > 0$

$\{c_{t+1}^t\}$ $\quad \dfrac{1}{c_{t+1}^t} = \lambda_{t+1} > 0$

$\{c_{t+2}^t\}$ $\quad \dfrac{1}{c_{t+2}^t} = \lambda_{t+2} > 0$

$\{s_t^t\}$ $\quad \lambda_t = \mu_t + \lambda_{t+1} R_{t+1} \rightarrow$ EE 1 $\qquad \dfrac{1}{c_t^t} = \dfrac{R_{t+1}}{c_{t+1}^t} + \mu_t$

$\{s_{t+1}^t\}$ $\quad \lambda_{t+1} = \mu_{t+1} + \lambda_{t+2} R_{t+2} \rightarrow$ EE 2 $\qquad \dfrac{1}{c_{t+1}^t} = \dfrac{R_{t+2}}{c_{t+2}^t} + \mu_{t+1}$

CS λ_t : $\lambda_t\left[w_t - c_t^t - s_t^t\right] = 0$; $\lambda_t \geq 0$ $\qquad$ μ_t : $\mu_t\left[\phi \frac{1}{R_t} w_{t+1} + s_t^t\right] = 0$; $\mu_t \geq 0$

λ_{t+1} : $\lambda_{t+1}\left[w_{t+1} + R_{t+1} s_t^t - c_{t+1}^t - s_{t+1}^t\right] = 0$; $\lambda_{t+1} \geq 0$ $\qquad$ μ_{t+1} : $\mu_{t+1}\left[s_{t+1}^t\right] = 0$; $\mu_{t+1} \geq 0$

λ_{t+2} : $\lambda_{t+2}\left[R_{t+2} s_{t+1}^t - c_{t+2}^t\right] = 0$; $\lambda_{t+2} \geq 0$

(a) Define a Competitive Equilibrium

Include all the relevant market clearing conditions.

① sequence of $\{R_t, w_t\}_{t=1}^{\infty}$

② sequence of HH choices $\{c_t^t, c_{t+1}^t, c_{t+2}^t, s_t^t, s_{t+1}^t\}_{t=1}^{\infty}$

c_1^0, c_1^{-1}, s_1^0

③ sequence of factor demands $\{K_t^d, N_t^d\}_{t=1}^{\infty}$ $[Y_t]$

s.t. (i) Given ①, ② HH max u

(ii) Given ①, ③ solves the firm max profit problem

(iii) Markets clear

Goods $\quad c_t^t + c_t^{t-1} + c_t^{t-2} + K_{t+1} \leq Y_t \quad \forall t$

Capital $\quad K_{t+1}^d \leq s_t^t + s_t^{t-1} \quad \forall t$

Labor $\quad N_t^d \leq 2$

(b) Borrowing Decision in Equilibrium

Without solving the problem but looking at FOC and market clearing

- Will young individuals ever borrow in equilibrium?

$\underline{BC}$

$c_t^t = w_t - s_t^t$

$c_{t+1}^t = w_{t+1} + s_t^t R_{t+1} - s_{t+1}^t$

$c_{t+2}^t = s_{t+1}^t R_{t+2}$

s_t

(1) $\frac{1}{c_t^t} = \frac{R_{t+1}}{c_{t+1}^t}$

(2) $\frac{1}{c_{t+1}^t} = \frac{R_{t+2}}{c_{t+2}^t}$

$> s_t^t \geq 0$

$> \boxed{s_t^t}$

Assume : $\mu_t = 0$

(1) $w_t R_{t+1} + s_{t+1}^t = w_{t+1} + 2 s_t^t R_{t+1}$

(2) $2 s_{t+1}^t = w_{t+1} + s_t^t R_{t+1}$

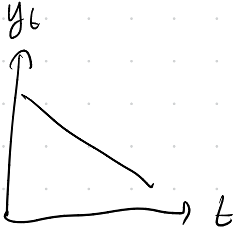
$s_t^t < 0$

(1) and (2)

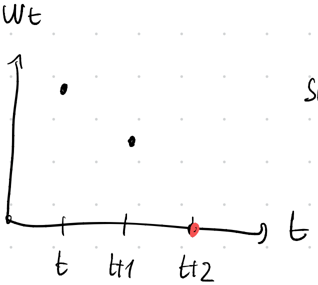
$$s_{t+1}^t = \frac{w_t R_{t+1} - w_{t+1}}{3}$$

$$\boxed{s_t^t = \frac{2}{3} w_t - \frac{5}{3} \frac{w_{t+1}}{R_{t+1}}}$$

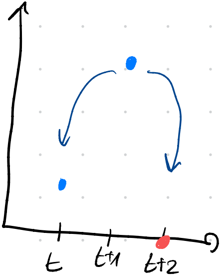
$\frac{2}{3} w_t < \frac{5}{3} \frac{w_{t+1}}{R_{t+1}}$



$S_t > 0$



$S_t > 0$



Review on Search: Mc Call Search Problem

Infinitely lived agent maximizes the expected present discounted value of income (i.e. the agent is risk-neutral)

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t y_t \right]$$

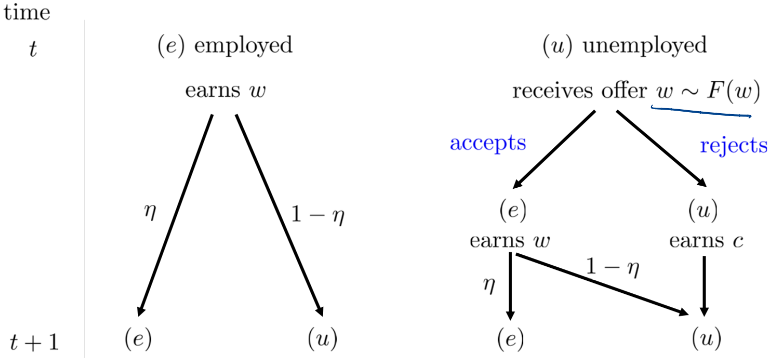
$\overbrace{u(c_t) = c_t}$ $= y_t$

- At any time t, the agent can be employed and supply 1 unit of labor at the wage w, or unemployed, and receive the unemployment benefit c.
- If unemployed, the agent receives each period t an offer in the form of a wage w from the distribution,

$$\text{Prob}(w \leq \hat{w}) = F(\hat{w}), \quad w \in [0, B].$$

- If accepts, becomes employed and receive the wage w from that period onward conditional on staying employed.
- If rejects, receive the benefit c, and enter the next period as still unemployed.
- An employed agent at t receives the wage w and keeps the job into next period with probability η, or loses it with probability $1 - \eta$. If that happens, the agent enters the next period, $t + 1$ as unemployed.
- Partial equilibrium model: Arrival of wage offer comes from exogenous distribution.

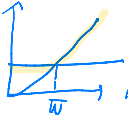
Review on Search: Timing is key!



$$V^e(w) = w + \eta\beta \underline{V^e(w)} + (1-\eta)\beta\bar{V}^u \rightarrow \bar{V}^u = \int V^u(w')\, F w'$$

$$V^u(w) = \max\left\{c + \beta\bar{V}^u, V^e(w)\right\}$$

Review on Search: Mc Call Search Problem



Steps for solving a problem like this:

- Decision of unemployed implies reservation wage:
  Accept if $w \geq \bar{w}$.
- $\bar{w}$ is derived by solving for the indifference between $c + \beta \bar{V}^u$ and $V^e(w)$:

$$V^u(w) = \max\left\{c + \beta \bar{V}^u, V^e(w)\right\}$$

$V^e(w) = c + \bar{V}^u$
↳ $\bar{w}$

- $\bar{w}$ also solves:

$$\bar{w} - c = H(\bar{w})$$

  - $\bar{w} - c$: Cost of searching one more time when offer $\bar{w}$ on hand.
  - $h(\bar{w})$: Expected benefit of searching one more time when offer $\bar{w}$ in hand.

$$\left.\begin{array}{ll} \text{cost}: & w - c \\ \text{benefit}: & H(w) \end{array}\right\} \bar{w}$$

Review on Search: Mc Call Search Problem

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Key result: MPS in $F \implies h(w; r') \geq h(w; r) \implies \bar{w}' \geq \bar{w}$.

- Even risk neutral agents behave as risk lovers!
- Reason: $F \to F'$ losses are limited, wins are higher.

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Model also solves for unemployment in steady state

$$u^* = \frac{1 - \eta}{1 - \eta + \eta(1 - \lambda)}$$

PS4 Problem B: On The Job Search

Workers maximize expected discounted income:

$$\sum_{t=0}^{\infty} \beta^t y_t, \quad \beta \in (0,1).$$

- Time is discrete, indexed by t.
- Workers can be employed or unemployed at the beginning of each period t.
- Unemployed:
  - Receive job offers by paying a search cost $z > 0$ at time t.
  - If an unemployed worker accepts an offer, they receive their first salary that period.
- Employed workers can search for new jobs
  - Pay cost z at time t to receive offer at time $t+1$.
  - If they accept, they receive the new salary at time $t+1$.
- From one period to the next, an employed worker can lose their job with probability η.

PS4 Problem B: On The Job Search

(a) Let $V(w)$ represent the value of the problem for a worker employed at salary w at the beginning of period t. Argue that the unemployed worker always searches and always accepts any offer she receive, so that the value for the unemployed worker before receiving an offer is:

$$V^u = \int_0^B V(w)dF(w) - z$$

(b) Using the value of the problem for the unemployed worker, write the recursive representation of the problem for the employed worker. Recall that the employed worker has to decide whether to search or not, and if searching, whether to accept or not the new job offer starting from next period.

(c) Use the functional equation derived in (b) to obtain expressions for:
   - The value of the problem for an employed worker that decides to search.
   - The value of the problem for an employed worker that decides not to search.

   Argue that the solution follows a reservation wage type: an employed worker at salary w will search if $w < \bar{w}$ and not search if $w \geq \bar{w}$.

(d) Derive an equation that equates the cost of one additional search to the expected benefit of an additional search at the reservation wage $\bar{w}$. Discuss how an increase in the cost of searching z affects $\bar{w}$, and how a mean-preserving spread in the distribution $F(w)$ affects $\bar{w}$.

PS4 B (a): Argue Unemployed Always Searches

Let $V(w)$ represent the value of the problem for a worker employed at salary w at the beginning of period t. Argue that the unemployed worker always searches and always accepts any offer she receive, so that the value for the unemployed worker before receiving an offer is:

$$V^u = \int_0^B V(w)dF(w) - z$$

The unemployed worker has to make two decisions. 1) Whether to search or not, 2) Wheter to affect or reject the offer.

- If the unemployed worker decides not to search, he/she will get 0 and again be in the unemployed status in the next period. Therefore, the value of not searching is βV^u.
- If the unemployed worker decides to search, he/she pays z and get a random offer w from $F(w)$ distribution. The agent now has to choose whether to accept or reject the offer.
  - If the agent rejects the offer, then he/she will get 0 and again be in the unemployed status in the next period.
  - If the agent accepts the offer, he/she will get $V(w)$.

When we express the above decision process,

$$V^u = \max\left\{\beta V^u, \int_0^B \max\{\beta V^u, V(w)\}dF(w) - z\right\}$$

PS4 B (a): Always searches and always accepts offer

$$V^u = \max\left\{\beta V^u, \int_0^B \max\{\beta V^u, V(w)\}dF(w) - z\right\}$$

1. First, consider $\max\{\beta V^u, V(w)\}$. There exists a certain level of $\bar{w}$ that satisfies $\beta V^u = V(\bar{w})$. To make our argument valid (always accept the offer), we need a assumption that
$$\beta V^u \leq V(0). \tag{1}$$
Then, we can rewrite the second term as follows.
$$\int_0^B \max\{\beta V^u, V(w)\}dF(w) - z = \int_0^B V(w)dF(w) - z$$

2. Now, consider $\max\{\beta V^u, \int_0^B V(w)dF(w) - z\}$. Suppose that that the agent decides not to search. Then,
$$V^u = \max\left\{\beta V^u, \int_0^B V(w)dF(w) - z\right\} = \beta V^u$$
This implies that $V^u = 0$. To make the agent always search, we need another assumption that
$$\int_0^B V(w)dF(w) - z \geq 0 \tag{2}$$

Under the two assumptions, we can argue that the agent will always search and accept the offer.

PS4 B (b): Recursive Representation of Employed Worker

The recursive representation of the employed worker is

$$
\begin{aligned}
V(w) &= \max\{V^{ns}(w), V^{s}(w)\} \\
&= \max\Bigg\{ w + \underbrace{(1-\eta)\beta V(w)}_{\textbf{(1)}} + \underbrace{\eta\beta V^{u}}_{\textbf{(2)}}, \\
&\qquad w - z + (1-\eta)\left[\underbrace{\beta \int_0^B \max\left\{V\left(w'\right), V(w)\right\} dF\left(w'\right)}_{(a)}\right] + \eta\left[\underbrace{\beta \int_0^B \max\left\{V\left(w'\right), V^{u}\right\} dF\left(w'\right)}_{(b)}\right] \Bigg\}
\end{aligned}
$$

- $V^{ns}(w)$ (Decide not to Search given the wage w)
  (1) Keep the current job
  (2) Lose the current job
- $V^{s}(w)$ (Decide to Search given the wage w)
  (a) Given that the agent does not lose the current job, decides whether to accept the new offer or not.
  (b) Given that the agent loses the current job, decides whether to accept the new offer or not.

Final Tips on Aiyagari

- I insist on checking our DS7 (210A Final question on Aiyagari), for intuition on what (1) the borrowing limit and (2) uncertainty and prudence in the utility function, imply for savings separately.
  - Main takeaway: In these framework, you need idiosyncratic risk and market incompleteness (borrowing limit) to generate precautionary savings.
  - Key: A borrowing limit on it's own will imply the directly constrained agents will borrow less (save more), but that $\neq$ precautionary savings.
- PS3 Problem B: Income Risk, Human Capital, and Limited Pledgeability of Returns is a good one for thinking how to generalize savings in other assets (like Human Capital).
  - Main takeaway: In order to characterize savings, your information source is the EE.
  - Key: The return of all assets will be equalized in equilibrium, so HK return is also r.

Final Advice and Farewells!

Final Advice:

- Feel 100% confident to send me emails until some hours before the exam.
- Prepare your Cheat-sheet in advance.
- Get your sleep and rest!

Farewells:

- It has been an amazing experience to be your TA for 210A and 210B.
- Maybe you don't realize now how much you've learned, but I see it!
- Keep up the good job and I'll be seeing you all the time!
  (especially if you choose Macro, yey!)