Econ 210A
Discussion Section N°9

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Plan for Today

- Time Constrained Optimal Strategy for your Final
  - Definitions
  - Tools
- Practice Question: Final 2022 - Dynamic Programming in Stochastic Models
- Practice Question: Final 2021 - Household Heterogeneity in the Neoclassical Model
- Follow up on SRCE - Computational Aiyagari

Past Final Exams and Time Constrained Strategy

Q1 has been about Dynamic Programming

- FE, prove $\exists$ and uniqueness in stochastic models (2021 Q1, 2022 Q1). Reference D2, more today.
- T/F (2024Q1). Solved in midterm review D5.
- 2023 Q1: Show FONC characterizes x^*. In your midterm solutions.

Q2-Q4 have focused on specific models

- Romer (2022 Q3, 2023 Q4, 2024 Q3). See D7.
- Ramsey (2022 Q2, 2023 Q2). Uploaded discussion.
- Aiyagari (2023 Q3, 2024 Q2). See D8.

Or deviations from Neoclassical growth model

- Structural change and non-homothetic preferences (2021 Q4, 2024 Q4)
- Household heterogeneity in the neoclassical model (2021 Q2). Today.

Past Final Exams and Time Constrained Strategy

I encourage you to study from the lecture slides and engage with all the material in the way that works best for you.

However, make sure you are extremely comfortable with the *basics* first:

- **Key Definitions**
  - BSP allocation
  - Decentralized equilibrium
- **Core Tools**
  - Translating a problem from the sequential to the recursive formulation
  - Solving constrained optimization problems to characterize the solution (EE, LL)

Your basics: Definitions you need to know

You will face at least one exercise that is either a:

- Benevolent Social Planner Problem
- Decentralized/Markets Problem

$\Longrightarrow$ Knowing these definitions is *free points*:

- Benevolent Social Planner allocation
  - Sequential
  - Recursive
- Competitive equilibrium
  - Sequence of Markets (SoM) Equilibrium
  - Recursive Competitive Equilibrium (RCE)

Definition: Benevolent Social Planner Allocation

- Sequential
  - $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$
  - maximizing lifetime utility s.t. resource constraint
- Recursive
  - $v(k)$ and policy functions $c(k)$, $n(k)$, $k'(k)$
  - maximizing lifetime utility s.t. resource constraint

Definition: Sequence of Markets (SoM) Equilibrium

A **SoM equilibrium** is a sequence of

1. Prices $\{r_t, w_t\}_{t=0}^{\infty}$
2. Household choices $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$
3. Firm choices $\{n_t^D, k_t^D\}_{t=0}^{\infty}$

such that

a. Given 1: 2 solves household UMP

$$\{c_t, n_t, k_{t+1}\} = argmax \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t)$$

st $c_t + k_{t+1} \leq r_t k_t + w_t n_t + (1-\delta)k_t + \pi_t$ in every $t = 0, 1, \ldots$

b. Given 1: 3 solves the firm profit max problem

$$\{k_t^D, n_t^D\} = argmax \quad \pi(k_t^D, n_t^D)$$

Where $\pi(k_t, n_t) = f(k_t, n_t) - r_t k_t - w_t n_t$ in all periods $t = 0, 1, \ldots$

c. Markets clear
   - Goods: $c_t + k_{t+1} = f(k_t, n_t) + (1-\delta)k_t$
   - Capital: $k_t = k_t^D$
   - Labor: $n_t = n_t^D$

   in all periods $t = 0, 1, \ldots$

Definition: Recursive Competitive Equilibrium (RCE)

A **RCE** is a set of functions on the household level and aggregate state variables k, K for

1. Prices $r(K), w(K)$
2. Household value $v(k, K)$, policy functions $c(k, K)$, $n(k, K)$, $k'(k, K)$, and an expectation for evolution of aggregate state $K' = G(K)$
3. Firm demands $N^D(K)$, $K^D(K)$

such that

a. Given 1: 2 solves household UMP

$$v(k, K) = \max_{c,n,k'} u(c, 1-n) + \beta v(k', K')$$
$$\text{s.t.} \quad c + k' - (1-\delta)k \leq r(K)k + w(K)n, \ K' = G(K)$$

b. Given 1: 3 solves the firm profit max problem

$$\{K^D(K), N^D(K)\} = argmax \quad f(K, N) - r(K)K - w(K)N$$

c. Markets clear
- Goods: $c + k' = f(K, N) + (1-\delta)k$
- Capital: $k = K^D$
- Labor: $n = N^D$

d. Consistency in expectation of law of motion for aggregate state $k = K \implies K' = G(K)$

Your basics: Tools you need to know

The optimization problem you will need to solve might be:

- A Benevolent Social Planner (BSP) problem
  - Sequential
  - Recursive
- An agent's problem (i.e. Household) in a decentralized equilibrium
  - Sequential
  - Recursive

$\Longrightarrow$ Be comfortable with these tools:

- Transform a Sequential Problem (SP) to a Recursive Formulation/Functional Equation/Bellman Equation (FE)
- Find the EE, LL. How?
  - (1) Manage your constraints. 2 alternatives:
    1. Replace constraints in the objective function
    2. Set a Lagrangian
  - (2) Using FONC, find EE and LL
    - SP: FONC $\Longrightarrow$ EE, LL
    - FE: FONC + Envelope $\Longrightarrow$ EE, LL

Examples

Definitions: BSP and Markets

2021 Q1 (≡2022 Q1) (A) Formulate the social planner's problem recursively. Define formally what is meant by the optimal policy that solves the social planner's allocation problem.

2021 Q4 (A) ... State the household problem, the firms' problems, and define the Sequence of Markets Equilibrium in this economy.

Tools: Characterize problem using EE, LL

2021 Q2 (A) Derive the conditions characterizing the social planners' (BSP) optimal allocation.

2021 Q3 (B) Set up the household problem and derive the Euler Equation and Labor-Leisure optimality conditions.

2021 Q4 (C) Derive the household optimality conditions ...

2022 Q3 (A) Characterize the household's optimal choices of consumption, capital, and labor.

2023 Q3 (A) ... Using the envelope condition, derive two formulas governing optimal household consumption-savings and labor supply decisions.

2023 Q4 (A) Characterize the household's optimal choices of consumption, capital, and labor.

Practice Question: Final 2022 - Dynamic Programming in Stochastic Models

Consider the neoclassical model with endogenous labor supply and stochastic TFP shocks. The social planner's problem is

$$\max_{c,k,n} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

subject to an aggregate resource constraint

$$c_t + k_{t+1} \leq (1-\delta)k_t + z_t k_t^{\alpha} n_t^{1-\alpha}$$

where z_t is an aggregate TFP shock, with finite support $z \in Z = \{z_1, \cdots, z_N\}$, that evolves following a first-order markov process $\pi(z_{t+1}|z_t) > 0$ for all t and $z \in Z$.

Practice Question: Final 2022 - Dynamic Programming in Stochastic Models

The social planner's problem is

$$\max_{c,k,n} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

s.t.

$$c_t + k_{t+1} \leq (1-\delta)k_t + z_t k_t^{\alpha} n_t^{1-\alpha}$$

where z_t is an aggregate TFP shock, with finite support $z \in Z = \{z_1, \cdots, z_N\}$, that evolves following a first-order markov process $\pi(z_{t+1}|z_t) > 0$ for all t and $z \in Z$.

(A) Formulate the social planner's problem recursively. Define formally what is meant by the optimal policy that solves the social planner's allocation problem.

(B) Prove that there exists a unique solution to the social planner's problem. If you need to make additional assumption, state them clearly.

(A) Formulate social planner's problem recursively

Sequential problem:

$$\max_{c,k,n} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

s.t.

$$c_t + k_{t+1} \leq (1-\delta)k_t + z_t k_t^{\alpha} n_t^{1-\alpha}$$

General key for SP $\rightarrow$ FE: Define what are the state x and choice y variables.
How can I be sure?

- Recall value function $V(x)$ and policy function $g(x) = y$ (determining optimal choice) both depend on the state variables
- Imagine how your EE would look like: can you write consumption (choice) as a function of all the states (policy function) from your EE?
  - If not, then you are missing a state variable!

Recursive problem:

$$\begin{aligned} V(K,z) &= \max_{C,N,K'} u(C,N) + \beta \cdot \mathbb{E}\left[V(K',z')\middle| z\right] \\ &= \max_{C,N,K'} u(C,N) + \beta \sum_{z'} \pi(z'|z) V(K',z') \\ s.t. \quad & C + K' \leq (1-\delta)K + zK^{\alpha}N^{1-\alpha} \end{aligned}$$

(A) Optimal policy that solves the social planner allocation problem

The Recursive Planner's Problem is:

$$
\begin{aligned}
V(K,z) &= \max_{C,N,K'} u(C,N) + \beta \cdot \mathbb{E}\left[V(K',z')|z\right] \\
&= \max_{C,N,K'} u(C,N) + \beta \sum_{z'} \pi(z'|z) V(K',z') \\
s.t. \quad & C + K' \leq (1-\delta)K + zK^{\alpha} N^{1-\alpha}
\end{aligned}
$$

Solution to this problem is given by:

- Value $V(K,z)$ that maximize the present value of lifetime utility at TFP z and capital level K.
- Policy functions $C(K,z)$, $N(K,z)$, $K'(K,z)$, which show the optimal choice of consumption, hours, and savings at TFP z and capital level K.

(B) Prove there exists a unique solution to BSP problem

- Sufficient assumptions are that $u(C, N)$ is increasing strictly concave and bounded in C; and decreasing strictly concave and bounded in N.
- Since $u(C, N)$ is strictly concave in C, then the budget constraint always holds with equality, hence: $C = (1-\delta)K + zK^{\alpha}N^{1-\alpha} - K'$.
- Let $f \geq g$ mean that, $(\forall x)(\forall y)$, $f(x, y) \geq g(x, y)$, i.e., the function is weakly dominates in either argument. Define the operator

$$T[f](K, z) \equiv \max_{K', N} u\left((1-\delta)K + zK^{\alpha}N^{1-\alpha} - K', N\right) + \beta \sum_{z'} \pi(z'|z) f(K', z')$$

- We need to show that $T[f]$ satisfies Blackwell's Sufficient Condition for a contraction.

(B) Blackwell's: Monotonicity

$$
\begin{aligned}
T[f](K,z) &\equiv \max_{K',N} u\left((1-\delta)K + zK^{\alpha}N^{1-\alpha} - K', N\right) + \beta \sum_{z'} \pi(z'|z) f(K',z') \\
&\geq \max_{K',N} u\left((1-\delta)K + zK^{\alpha}N^{1-\alpha} - K', N\right) + \beta \sum_{z'} \pi(z'|z) g(K',z') \\
&= T[g](K,z)
\end{aligned}
$$

where the inequality holds because, for every point (K', z'), $f(K', z') \geq g(K', z')$ and the probabilities $\pi(z'|z)$ enter linearly.

$\implies T[f](K,z)$ satisfies monotonicity

(B) Blackwell's: Discounting

$$
\begin{aligned}
T[f+a](K,z) &= \max_{K',N} u\left((1-\delta)K + zK^{\alpha}N^{1-\alpha} - K', N\right) + \beta \sum_{z'} \pi(z'|z)\left[f(K',z') + a\right] \\
&= \max_{K',N} u\left((1-\delta)K + zK^{\alpha}N^{1-\alpha} - K', N\right) + \beta \sum_{z'} \pi(z'|z) f(K',z') \\
&\quad + \beta \sum_{z'} \pi(z'|z) a \\
&= T[f](K,z) + \beta a
\end{aligned}
$$

where the last equality follows from the fact that a is a constant and $\sum_{z'} \pi(z'|z) = 1$.

$\implies T[f](K,z)$ satisfies discounting.

(B) Uniqueness of solution to the BSP problem

Note

- After conditioning on z, this is a stationary problem.
- For a particular level of z and a particular level of K, there is a unique choice of (K', N) that maximize (expected) utility.

Practice Question: Midterm 2022 - Household Heterogeneity in the Neoclassical Model

Consider a population of households $i \in [0,1]$ with separable preferences over consumption c_t^i and labor n_t^i given by,

$$\max_{\{c_t^i, n_t^i\}_{\forall t}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[u(c_t^i) - \nu(n_t^i) \right]$$

where utility function $u(\cdot)$ satisfies neoclassical assumptions and $\nu(\cdot)$ is increasing and strictly convex, capturing the disutility of labor. The household budget constraint is given by

$$c_t^i = w_t a_t^i n_t^i$$

where w_t is the (effective) wage rate and a_t^i is household i's stochastic labor productivity, distributed i.i.d. over time and across households.

(a) Characterize household i's optimal consumption and labor supply choices. Describe how each household's decisions varies with its idiosyncratic labor productivity.

Practice Question: Household Heterogeneity in the Neoclassical Model

(b) Consider the Social Planner who maximizes aggregate household utility given welfare weights θ_i. Formally, the social planner solves

$$\max_{\{c_t^i, n_t^i\}_{\forall t,i}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \sum_i \theta^i \times \left[u(c_t^i) - \nu(n_t^i) \right]$$

subject to an aggregate resource constraint,

$$\sum_i c_t^i = \sum_i w_t a_t^i n_t^i$$

Derive the first order conditions of the social planner governing the allocation of consumption and labor in the aggregate economy.

(c) How do the relative hours worked for households i and j (n_t^i / n_t^j) vary with their productivity shocks under the BSP allocation? Provide economic intuition for the result.

(d) How do the relative consumption levels of household i and j (c_t^i / c_t^j), vary with their income shocks under the BSP allocation? Provide economic intuition for the result.

(e) How does the allocation chosen by the social planner compare to the famous communist maxim of Karl Marx (1865) "*To each according to their need; from each according to their ability*"?

(f) How does the social planner's allocation compare to the decentralized equilibrium in part (a)? Is the decentralized equilibrium efficient? Explain your answer.

(a) Characterize household optimal consumption and labor supply decision

Household i solves

$$\max_{\{c_t^i, n_t^i\}_{\forall t}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[u(c_t^i) - \nu(n_t^i) \right]$$

$$\text{s.t.} \quad c_t^i = w_t a_t^i n_t^i \qquad \forall\ t$$

Lagrangian

$$\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left[u(c_t^i) - \nu(n_t^i) \right] + \lambda_t^i (w_t a_t^i n_t^i - c_t^i) \right\}$$

FOC ($\forall\ t$)

$$\{c\}: \quad \frac{\partial \mathcal{L}}{\partial c_t^i} = 0 \quad \leftrightarrow \quad u'(c_t^i) = \lambda_t^i$$

$$\{n\}: \quad \frac{\partial \mathcal{L}}{\partial n_t^i} = 0 \quad \leftrightarrow \quad \nu'(n_t^i) = \lambda_t^i w_t a_t^i$$

Labor Leisure equation: $\nu'(n_t^i) = u'(c_t^i) w_t a_t^i$

(a) Describe how each household's decision vary with idiosyncratic labor productivity

Want to understand how a affects n and c.

We start with n. Replacing budget constraint $c_t^i = w_t a_t^i n_t^i$ in LL:

$$\text{LL:} \qquad \nu'(n_t^i) = u'(w_t a_t^i n_t^i) w_t a_t^i$$

Rewrite the LL equation:

$$F(n,a) = \nu'(n) - u'(wan)\, wa = 0$$

By the Implicit Function Theorem:

$$\frac{\partial n}{\partial a} = -\frac{F_a(n,a)}{F_n(n,a)}$$

Partial derivatives:

$$\begin{aligned} F_a(n,a) &= -\left(w\, u'(wan) + w^2 an\, u''(wan)\right) \\ F_n(n,a) &= \nu''(n) - w^2 a^2\, u''(wan) \end{aligned}$$

Implicit derivative:

$$\frac{\partial n}{\partial a} = \frac{w\, u'(wan) + w^2 an\, u''(wan)}{\nu''(n) - w^2 a^2\, u''(wan)}$$

(a) Describe how a affects n

Implicit derivative:

$$\frac{\partial n}{\partial a} = \frac{w\, u'(wan) + w^2 an\, u''(wan)}{\nu''(n) - w^2 a^2\, u''(wan)}.$$

Given $u' > 0$, $u'' < 0$, $\nu'' > 0$:

- Denominator is strictly positive:

$$\nu''(n) - w^2 a^2 u''(\cdot) = \nu''(n) + w^2 a^2 |u''(\cdot)| > 0.$$

- Numerator has ambiguous sign:

$$w\left[u'(wan) + wan\, u''(wan)\right] \quad \text{can be positive or negative.}$$

a is acting like an increase in wage/decrease in relative price of consumption.
Sign depends on which effect dominates:

- **Substitution**: relative price of consumption to leisure decreases $\rightarrow$ subs. leisure to consumption ($\uparrow n$)
- **Wealth**: agent is wealthier over all, inducing the desire to increase both consumption and leisure ($\downarrow n$)

(a) Describe how a affects n: Illustrative Example

Recall Discussion 5: Endogenous Labor Supply in NGM - Midterm 2021.

The functional forms were:

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \qquad \nu(n) = \psi \frac{n^{1+\phi} - 1}{1+\phi}$$

Find $u'(c)$ and $u''(c)$:

$$u'(c) = c^{-\gamma} \qquad u''(c) = -\gamma c^{-\gamma-1}$$

Replace in numerator to find sign of $\partial n / \partial a$

$$\begin{aligned}
\text{numerator} &= w\left[\, u'(wan) \; + \; wan\, u''(wan) \,\right] \\
&= w\left[(wan)^{-\gamma} - \gamma \; wan\,(wan)^{-\gamma-1}\right] \\
&= w\left[(wan)^{-\gamma} - \gamma \; (wan)^{\gamma}\right] \\
&= w(wan)^{-\gamma}[1-\gamma]
\end{aligned}$$

- If $\gamma > 1$: $\partial n / \partial a > 0$ - substitution effect dominates
- If $\gamma < 1$: $\partial n / \partial a < 0$ - wealth effect dominates
- If $\gamma = 1$: $\partial n / \partial a = 0$ - substitution and wealth effect cancel out (what we impose in D5 for BGP)

(a) Describe how a affects c

Now define $$J(a,c) = v'\left(\frac{c}{wa}\right) - u'(c)\, wa = 0.$$

By the Implicit Function Theorem:

$$\frac{\partial c}{\partial a} = -\frac{J_a(a,c)}{J_c(a,c)}.$$

Partial derivatives:

$$J_a(a,c) = v''\left(\frac{c}{wa}\right)\left(-\frac{c}{wa^2}\right) - w\, u'(c), \qquad J_c(a,c) = v''\left(\frac{c}{wa}\right)\left(\frac{1}{wa}\right) - wa\, u''(c).$$

Implicit derivative:

$$\frac{\partial c}{\partial a} = \frac{\frac{c}{wa^2}\, v''\left(\frac{c}{wa}\right) + w\, u'(c)}{\frac{1}{wa}\, v''\left(\frac{c}{wa}\right) - wa\, u''(c)} > 0$$

- Denominator:

$$\frac{1}{wa}\, v''\left(\frac{c}{wa}\right) - wa\, u''(c) = \frac{1}{wa}\, v''(\cdot) + wa\, |u''(c)| > 0.$$

- Numerator:

$$\frac{c}{wa^2}\, v''\left(\frac{c}{wa}\right) + w\, u'(c) > 0.$$

(b) Derive the FOC of the social planner

BSP solves

$$\max_{\{c_t^i, n_t^i\}_{\forall t,i}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \sum_i \theta^i \times \left[u(c_t^i) - \nu(n_t^i) \right]$$

$$\text{s.t.} \quad \sum_i c_t^i = \sum_i w_t a_t^i n_t^i \quad \forall t$$

Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ \sum_i \theta_i \left[u(c_t^i) - \nu(n_t^i) \right] + \lambda_t \sum_i \left[w_t a_t^i n_t - c_t^i \right] \right\}$$

FOC ($\forall\ i, t$)

$$\{c\} : \quad \frac{\partial \mathcal{L}}{\partial c_t^i} = 0 \quad \leftrightarrow \quad \theta_i u'(c_t^i) \quad = \lambda_t$$

$$\{n\} : \quad \frac{\partial \mathcal{L}}{\partial n_t^i} = 0 \quad \leftrightarrow \quad \theta_i \nu'(n_t^i) \quad = \lambda_t w_t a_t^i$$

(c) Relative Hours in the BSP Allocation

FOC under BSP:

$$\theta_i \nu'(n_t^i) = \lambda_t w_t a_t^i$$

Take the ratio for i and j:

$$\frac{\nu'(n_t^i)}{\nu'(n_t^j)} = \left(\frac{a_t^i}{a_t^j}\right)\left(\frac{\theta_i}{\theta_j}\right)^{-1}$$

Since $\nu'' > 0$, $\nu'(n)$ is increasing, so:

$$\frac{n_t^i}{n_t^j} \text{ increases in } \frac{a_t^i}{a_t^j} \qquad \frac{n_t^i}{n_t^j} \text{ decreases in } \frac{\theta_i}{\theta_j}$$

Intuition:

- **Productivity shocks a:** More productive households generate more output per hour, so the planner assigns them more labor.
- **Weights θ:** Households with higher utility weight θ_i work less.

(d) Relative Consumption in the BSP Allocation

FOC for consumption under BSP:

$$u'(c_t^i) = \lambda_t \, \theta_i^{-1}$$

Take the ratio for i and j:

$$\frac{u'(c_t^i)}{u'(c_t^j)} = \left(\frac{\theta_i}{\theta_j}\right)^{-1}$$

Since $u'' < 0$, $u'(c)$ is decreasing, so:

$$\frac{c_t^i}{c_t^j} \text{ increases in } \frac{\theta_i}{\theta_j}, \qquad \text{and is independent of } \frac{a_t^i}{a_t^j}.$$

Intuition:

- **Productivity shocks a:** Do not affect relative consumption. The planner provides full insurance.
- **Weights θ:** Higher welfare weight θ_i lowers $u'(c_t^i)$, hence raises c_t^i.

(e) Marx's Maxim and the BSP Allocation

Marx (1865):

"To each according to their need; from each according to their ability."

Implication of BSP with equal weights $(\theta_i = \theta_j)$:

- **Hours:** More productive workers (a_t^i high) optimally work more.
- **Consumption:** All individuals consume the same amount.

Conclusion: The BSP allocation is consistent with Marx's maxim: workers contribute according to ability and receive (consume) according to need.

(f) Efficiency: BSP vs. Decentralized Equilibrium

BSP allocation:

- By construction, it is **Pareto efficient**. It maximizes social welfare given the social welfare function specified and aggregate resource constraints.

Decentralized equilibrium:

- **Not efficient** due to incomplete markets: households cannot insure against idiosyncratic productivity shocks.
- Missing insurance markets imply (here) consumption differs across individuals even when society would prefer equal consumption (with equal weights).

Implication: With full insurance markets, the decentralized equilibrium would replicate the BSP allocation with equal weights — all individuals would consume the same amount.

Follow up on SRCE - Computational Aiyagari

Stationary Recursive Competitive Equilibrium (SRCE)

Solution for the SRCE is finding λ^* and r^* s.t.:

$$\lambda^* = \mathcal{H}(\lambda^*) \tag{1}$$

$$K(r^*) = A(r^*; \lambda^*) \tag{2}$$

Numerical Solution to the SRCE: Parametrization in PS4

- **Utility function:**

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

- **Production function:**

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

- **Labor income risk:**
  - Each period, the agent is in a state $s_t \in \{0.05, 1\}$.
    - $s_t = 0.05$ (low productivity state).
    - $s_t = 1$ (high productivity state).
  - Labor income in period t is $s_t w$, where w is the equilibrium wage rate.
- **Agent's state evolution:**
  - Follows a Markov chain with transition matrix $\mathcal{P} = \begin{pmatrix} \pi(0.05|0.05) & \pi(1|0.05) \\ \pi(0.05|1) & \pi(1|1) \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.8 \end{pmatrix}$.
- **Parameter values:**
  - $\alpha = 0.3$, $\delta = 0.08$, $\beta = 0.98$, $\sigma = 1.5$.

Numerical Solution to the SRCE: Steps

1. Pick a K, given $N = \Pi S$, compute r and w
   - from FONC of firms
2. Solve household's problem under r and w
   - Recall Blackwell + CMT provides an algorithm to find HH solution: VFI
   - Guess V_0 and iterate with the Bellman operator until convergence $TV^* = V^*$
   - Once converged, plug state in V^* and solve FONC for *policy function g*
   - This gives you a' and c
3. Use solution to household problem to compute stationary measure λ^* via $\mathcal{H}$
4. Compute $E[a(r)]$ and determine excess demand for capital

$$K - E[a(r)]$$

   - K being your initial guess for K
5. If excess demand is zero, equilibrium found; if not, adjust K and repeat 1-4.

SRCE

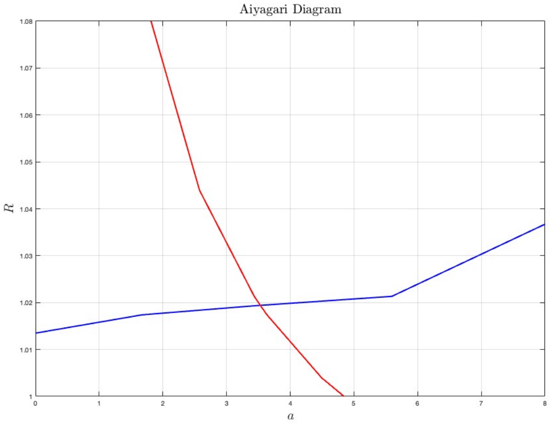

Figure 1: Capital market equilibrium in SRCE

Distribution of agents $\lambda(a, s) \to$ Wealth distribution (a)